

Pre board- 3 EXAMINATION 2023-24

APPLIED MATHEMATICS (241)

Class: XII Commerce

Duration: 3 Hrs.

Date: 10-01-2023

MARKING SCHEME

Max. Marks: 80

Question	Answer	Scheme
1	When $a \equiv b \pmod{n}$ implies $a - b = kn$ for some $n \in \mathbb{N}$. Here, $x - 17 = k \times 4$ and $x > 18$ and ≤ 25 . Therefore, the least value that satisfies this statement is $x = 21$	Answer B
2	Given $31 \equiv b \pmod{5}$. this means $31 - b = 5 \times k$. Here, $31 - 21 = 5 \times 2, 31 - 31 = 5 \times 0$ and $31 - 16 = 5 \times 3$. Therefore, all these statements are satisfied by b.	Answer D
3	Total number of students = 1000. Let the number of girls be x . Therefore, $70\% \text{ of } (1000 - x) + 80\% \text{ of } x = 76\% \text{ of } 1000$ Hence $7000 - 70x + 80x = 76000$ or $10x = 6000$ or $x = 600$.	Answer B
4	Amount that pipe A can fill in 1 minute $= \frac{1}{12}$. Amount that pipe B can fill in 1 minute $= \frac{1}{18}$. When both pipes are open, Amount that filled in 1 minute $= \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$ Therefore, time taken to fill the tank is $\frac{36}{5} = 7.2$ minutes	Answer C
5	$I = \int \frac{x^3 + 2x - 2}{x+5} dx = \int 1 - \frac{2}{x+5} dx = x - 2 \log(x+5) + c$	Answer B
6	Given $y = x$. Now $\frac{dy}{dx} = 1$ and further $\frac{d^2y}{dx^2} = 0$	Answer B
7	Property: sum of all probabilities of elementary events = 1 Therefore, $0.1 + k + 2k + 2k + k = 1$ or $6k = 0.9$ and so $k = \frac{0.9}{6} = \frac{9}{60} = 0.15$	Answer C
8	A die is thrown 6 times. Let X be a random variable of getting an even number. Probability of success $= \frac{1}{2}$. Therefore, $P(x = 2) = 6C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4 = \frac{15}{64}$	Answer B
9	Mean of a random variable $= np$ and Variance of the random variable is npq . Given $n = 4$ and $p = \frac{1}{3}$. Therefore, mean $= \frac{4}{3}$ and Variance $= \frac{8}{3}$	Answer D
10	A set of observations recorded at an equal interval of time is called Time Series data	Answer D
11	The number of components does a time series data have = 3	Answer C
12	Mean (μ_a) before campaign = Mean (μ_b) after campaign	Answer A
13	Sum of squares of residuals or errors must be MINIMUM	Answer C
14	Present value of a sequence of payments made at the end of each 6 months @ Rs.60 is $= P = \frac{R}{i} = \frac{60}{0.02} = Rs. 3000$	Answer A
15	The common shaded region taking all constraints on a graph sheet is known as Feasible region	Answer D

16	Annual depreciation = (cost of the machine – scrap value) / number of life years. Therefore, Annual depreciation = $\frac{40000-8000}{4} = \frac{32000}{4} = \text{Rs. } 8000$	Answer A
17	The total number of samples = 50 bulbs. 15 bulbs are Bajaj, 17 bulbs are Surya and 18 bulbs are Crompton. The point estimate of population proportion of Surya = $\frac{17}{50} = \frac{34}{100} = 0.34$	Answer B
18	Investment on one share of worth Rs.100 is Rs.80. Rate of dividend = 12% Therefore, effective rate of return ERR = $\frac{\text{return}}{\text{investment}} \times 100 = \frac{12}{80} \times 100 = 15\%$	Answer C
19	Assertion: TRUE. Demand raises on the respective seasons. Reason: TRUE. Seasonal changes are considered within a period of an year	Answer A
20	$R(x) = 3x^2 + 36x + 5$. Therefore, marginal revenue = $\frac{dR}{dx} = 6x + 36$. The marginal revenue when $x = 5$ is $6(5) + 36 = 66$. So, Assertion is TRUE. Reason: TRUE and reason supports the assertion statement	Answer A

SECTION – B VERY SHORT ANSWER 2 marks each

21	Data given are 5, 8, 10, 7, 10, 14. The point estimation of the population mean = Sum of all observations/number of observations = $\frac{54}{6} = 9$ The point estimate of the population standard deviation = $\sqrt{\frac{\sum(x - M)^2}{n - 1}} = \sqrt{\frac{16 + 1 + 1 + 4 + 1 + 25}{5}} = \sqrt{9.6} = 3.09$	1 0.5 0.5
22	Let the number of model A products = x and the number of model B products = y. Now the objective function $z = 8000x + 12000y$ subject to the constraints $9x + 12y \leq 180$ and $x + 3y \leq 30$. The corner points obtained on the graph are (20,0), (12, 6) and (0, 10).	1 1
23	Area of a triangle = $\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$ $= \frac{1}{2}\{3(2 - 1) - 4(1 - 8) + 5(8 - 2)\} = \frac{1}{2}\{3 + 28 + 30\} = \frac{61}{2} = 30.5$ square units	0.5 1 0.5
OR	Given $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$. Determinant of A = $2 \times 3 - 4 \times (-2) = 14 \neq 0$. Inverse of A exists. Adjugate of A = $\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$. Hence $A^{-1} = \frac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$	1 1
24	A defeats B by 60 metres in a race = A defeats B by 12 seconds. Therefore, A runs the race of 60 metres in 12 seconds. So, he takes 1 metre in 1/5 seconds. Total distance covered in the race = 500 metres The time A has taken to run 500 metres = $500 \times 1/5 = 100$ seconds	0.5 0.5 0.5 0.5
OR	A pump fills a tank in 2 hours. Let a leak empties the tank in x hours. Therefore, quantity of tank filled in 1 hour = $\frac{1}{2} - \frac{1}{x} = \frac{3}{7}$ or $\frac{x-2}{2x} = \frac{3}{7}$ or $7x - 14 = 6x$ or $x = 14$ Hence the leakage empties the tank in 14 hours if kept open alone.	1 1
25	Present value of perpetuity $P = \frac{R}{i}$ or $i = \frac{R}{P}$ or $\frac{r}{100} = \frac{R}{P} = \frac{500}{10000}$. therefore, $r = 10\%$	1 1

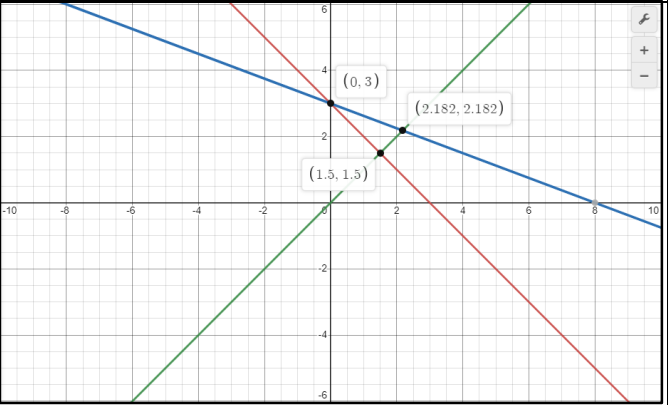
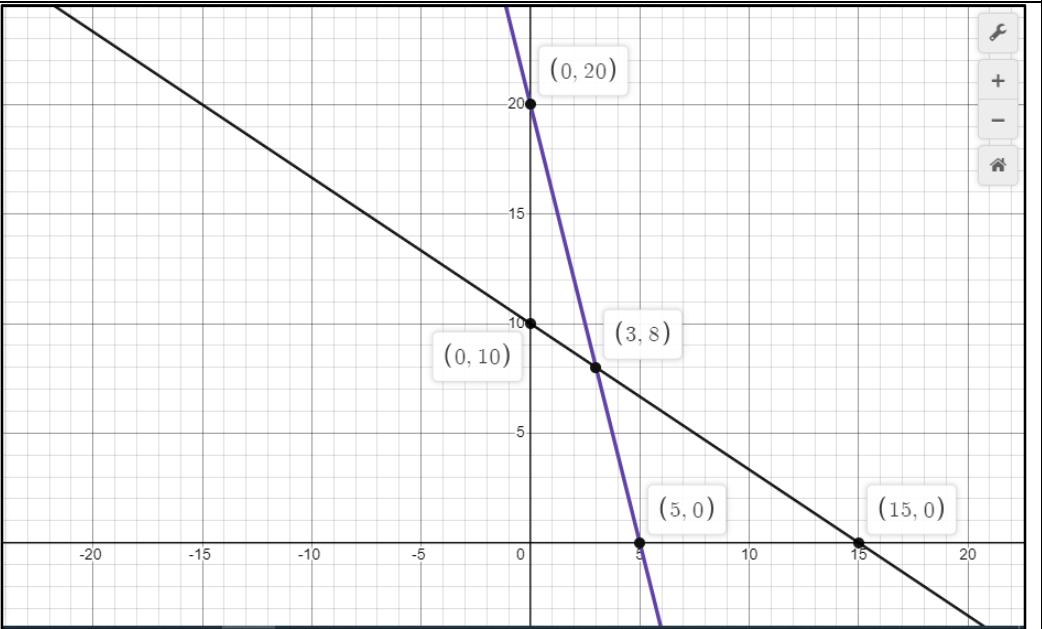
SECTION – C SHORT ANSWER 3 marks each

26	Cost of new house = Rs.4500000; down payment = Rs.500000; loan = Rs.4000000 Therefore, $EMI = \frac{P \times i \times (1+i)^n}{(1+i)^n - 1}$ $EMI = \frac{4000000 \times 0.005 \times (1.005)^{300}}{(1.005)^{300} - 1} = \frac{20000 \times 4.4650}{3.4650} = \frac{89300}{3.4650} = \text{Rs. } 25772$	0.5 0.5 0.5 0.5 1
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27	Apply partial fraction: $\frac{2x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$; we simplify $2x + 1 \equiv A(x - 2) + B(x + 1)$ $\int \frac{1}{x-2} + \frac{5}{x+1} dx = \frac{1}{3} \log(x-2) + \frac{5}{3} \log(x+1) + c$	1 1 1
OR	Apply partial fraction of integration: $\frac{3x-2}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$. Therefore, $3x - 2 \equiv A(x - 2)^2 + B(x + 1)(x - 2) + C(x + 1)$. Find A, B and C values $\int \frac{-5}{x+1} + \frac{5}{x-2} + \frac{4}{(x-2)^2} dx = -\frac{5}{9} \log(x+1) + \frac{5}{9} \log(x-2) - \frac{8}{(x-2)} + c$	0.5 0.5 0.5 0.5 1
28	Sinking fund amount = Rs.1000000; number of years = 10; r = 5% compounded semi annually. Monthly deposit = $R = \frac{A \times i}{(1+i)^n - 1} = \frac{1000000 \times 0.025}{(1.025)^{20} - 1} = \frac{25000}{1.6386 - 1} = Rs. 39148$	1 1 1
29	Definite integral: $\int_1^4 x - 5 dx = \int_1^4 -x + 5 dx = -\frac{x^2}{2} + 5x = -8 + 20 + \frac{1}{2} - 5 = 7.5$ Because of the turning point is 5, the given interval is below it and hence it has only one definite integral part.	1 1 1
OR	The demand function $D(x) = 100 - 8x$. Given when $p_0 = 4$, therefore $4 = 100 - 8x_0$. so, $x_0 = 12$. Therefore, consumer surplus = $\int_0^{x_0} D(x) dx - x_0 p_0 =$ $\int_0^{12} 100 - 8x dx - 12 \times 4 = 100x - \frac{8x^2}{2} - 48 = 1200 - 4 \times 144 - 48 = 576$	1 1 1
30	The equation formed are: $4x + 3y + 2z = 60$; $2x + 4y + 6z = 90$ and $6x + 2y + 3z = 70$ The coefficient matrix = $\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$ and $\Delta = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 10$ Cofactor matrix of A = $\begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}$. The solution of the linear equations: $X = A^{-1} \times B$ $= \frac{1}{10} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \times \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$. Solution are $x = Rs. 25$; $y = Rs. 40$; $z = Rs. 40$	1 1 1
31	Given $y = x^3 \log x$. First derivative: $\frac{dy}{dx} = x^3 \times \frac{1}{x} + \log x \times 3x^2 = x^2 + 3x^2 \log x$ Second derivative: $\frac{d^2y}{dx^2} = 2x + 3x^2 \times \frac{1}{x} + \log x \times 6x = 2x + 3x + 6x \log x = 5x + 6x \log x$	1 1 1

SECTION – D LONG ANSWER TYPE 5 marks each

32	Volume of the tank = 4000 cubic cm. $V = x^2 y$ or $y = \frac{4000}{x^2}$ Surface area of the tank with square bottom = $S = x^2 + 4xy = x^2 + 4x \times \frac{4000}{x^2}$ $= \frac{ds}{dx} = 2x + 16000 \times -\frac{1}{x^2} = 2x - \frac{16000}{x^2}$. If $\frac{ds}{dx} = 0$ then $x^3 = 8000$ and so $x = 20$ cm Therefore, dimensions of the tank are $l = 20$ cm, $b = 20$ cm and $h = 20$ cm	1 1 1 1 1
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OR	<p>The equation of the curve is $y^2 = 2x$ and the point is $(1, 4)$. Differentiating with respect to x, we get $2y \frac{dy}{dx} = 2$ or $\frac{dy}{dx} = \frac{1}{y}$. Therefore, slope of the normal = $-y$. Now equation of the line passing through $(1, 4)$ with slope $-y$ is $y - 4 = -y(x - 1)$. Now solving the curve we get $y - 1 = -y \left(\frac{y^2}{2} - 1 \right)$ or $2y - 8 = -y^3 + 2y$ or $y^3 = 8$ and hence $y = 2, x = 2$. Therefore, the nearest point is $(2, 2)$</p>	1 1 1 1 1
33	<p>Let $X =$ age among adults in a region; $\mu = 8; \sigma = 5. n =$number of adults = 100 For $x = 10, z = \frac{10 - \mu}{\sigma} = \frac{10 - 8}{5} = \frac{2}{5} = 0.4$ For $x = 14, z = \frac{14 - \mu}{\sigma} = \frac{14 - 8}{5} = \frac{6}{5} = 1.2$ therefore, $P(10 < X < 14) = P(0.4 < Z < 1.2) = F(1.2) - F(0.4) = 0.8849 - 0.6554 = 0.2295$ Number of adults in the age group between 10 and 14 years = $0.2295 \times 100 = 22.95 \approx 23$ $P(X > 14) = P(Z > 1.2) = 1 - P(Z \leq 1.2) = 1 - 0.8849 = 0.1151$ Number of adults in the age more than 14 years = $0.1151 \times 100 = 11.51 \approx 12$ adults</p>	1 1 1 1 1
OR	<p>Poisson distribution: probability of a bucket produced is defective $p = 0.03$ Number of buckets produced $n = 100$. Therefore, $\lambda = np = 100 \times 0.03 = 3$ $P(\text{zero defective bucket}) = P(X = 0) = \frac{e^{-3} \times 3^0}{0!} = \frac{0.049 \times 1}{1} = 0.049$ $P(\text{at most one defective bucket}) = P(X \leq 1) = P(0) + P(1) = 0.049 + \frac{e^{-3} \times 3^1}{1!}$ Therefore, $P(X \leq 1) = 0.049 + 0.049 \times 3 = 0.049 + 0.147 = 0.196$</p>	1 1 1 1 1
34	<p>At $(0, 3), z = 22 \times 0 + 44 \times 3$ $Z = 132$ At $(1.5, 1.5), z = 22 \times 1.5 + 44 \times 1.5$ $Z = 33 + 66 = 99$ At $(2.182, 2.182),$ $Z = 22 \times 2.182 + 44 \times 2.182$ $= 48.004 + 96.008 = 144.012$ Maximum of $z = 144$. The solution region is bounded between $(0, 3), (1.5, 1.5)$ and $(2.182, 2.182)$</p> 	1 1 1 1 1
OR	 <p>At $(15, 0),$ $Z = 18 \times 15 + 10 \times 0 = 270$ At $(3, 8), z = 18 \times 3 + 10 \times 8 = 54 + 80 = 134$ At $(0, 20), z = 18 \times 0 + 10 \times 20 = 200$ Minimum of these z values is 134 and at $(3, 8)$.</p>	1 1 1 1 1

35	<p>The linear equations formed are: $x + y + z = 20$; $2x + y - z = 23$; $3x + y + z = 46$.</p> <p>Determinant of coefficients = $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{vmatrix} = (1(1 + 1) - 1(2 + 3) + 1(2 - 3)) = -4$</p> <p>Now $\Delta_1 = \begin{vmatrix} 20 & 1 & 1 \\ 23 & 1 & -1 \\ 46 & 1 & 1 \end{vmatrix} = 20(1 + 1) - 1(23 + 46) + 1(23 - 46) = 40 - 69 - 23 = -52$</p> <p>Now $\Delta_2 = \begin{vmatrix} 1 & 20 & 1 \\ 2 & 23 & -1 \\ 3 & 46 & 1 \end{vmatrix} = 1(23 + 46) - 20(2 + 3) + 1(92 - 69) = 69 - 100 + 23 = -8$</p> <p>Now $\Delta_3 = \begin{vmatrix} 1 & 1 & 20 \\ 2 & 1 & 23 \\ 3 & 1 & 46 \end{vmatrix} = 1(46 - 23) - 1(92 - 69) + 20(2 - 3) = 23 - 23 - 20 = -20$</p> <p>Therefore, $x = \frac{\Delta_1}{\Delta} = -\frac{52}{-4} = 13$; $y = \frac{\Delta_2}{\Delta} = -\frac{8}{-4} = 2$; $z = \frac{\Delta_3}{\Delta} = -\frac{20}{-4} = 5$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
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SECTION – E CASE STUDY QUESTIONS 4 marks each

36a	<p>The amount filled when pipe A and pipe B are both opened, in 1 hour = $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$</p> <p>Time taken to fill, when both pipe A and B are opened is $\frac{24}{7} = 3\frac{3}{7}$ hours</p>	<p>1</p>
36b	<p>The amount filled when pipe A and pipe C are both opened, in 1 hour = $\frac{1}{6} - \frac{1}{12} = \frac{1}{12}$</p> <p>Time taken to fill, when both pipe A and C are opened is 12 hours</p>	<p>1</p>
36c	<p>The amount filled when pipe B and C are both opened, in 1 hour = $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$</p> <p>Time taken to fill, when both pipe B and C are opened is 24 hours</p>	<p>1</p> <p>1</p>
OR	<p>The amount filled when all three pipes are opened, in 1 hour = $\frac{1}{6} + \frac{1}{8} - \frac{1}{12} = \frac{5}{24}$</p> <p>Time taken to fill, when all three pipes are opened is $\frac{24}{5} = 4\frac{4}{5}$ hours</p>	<p>1</p> <p>1</p>
37a	<p>Sum of all probabilities = 1. Therefore, $0.1 + k + 2k + 2k + k = 1$ or $k = \frac{0.9}{6} = 0.15$</p>	<p>0.5</p> <p>0.5</p>
37b	<p>$P(\text{studies for three hours}) = P(x = 3) = k(5 - 3) = 2k = 2 \times 0.15 = 0.3$</p>	<p>1</p>
37c	<p>$P(\text{studies for two hours}) = P(x = 2) = k \times 2 = 2 \times 0.15 = 0.3$</p>	<p>1</p> <p>1</p>
OR	<p>$P(\text{studies at least two hours}) = P(x \geq 2) = 1 - P(x < 2) = 1 - \{P(0) + P(1)\}$ $= 1 - \{0.1 + 0.15\} = 0.75$</p>	<p>1</p> <p>1</p>
38a	<p>The forecast for the year 2006 for Urban group is $y = 23 + 6.9 \times 2006$ $= 23 + 13841.4 = 13864.40$</p>	<p>0.5</p> <p>0.5</p>
38b	<p>The forecast for the year 2006 for Rural group is $y = 11.6 + 5.2 \times 2006$ $= 11.6 + 5.2 \times 2006 = 11.6 + 10431.2 = 10442.80$</p>	<p>0.5</p> <p>0.5</p>
38c	<p>The trend line by the method of least squares for Rural Indians group Is $y = 11.6 + 5.2x$</p>	<p>1</p> <p>1</p>
OR	<p>The trend line by the method of least square for Urban Indians group Is $y = 23 + 6.9x$</p>	<p>1</p> <p>1</p>